CONSTRAINED CONJUNCTIVE-USE FOR ENDOGENOUSLY SEPARABLE WATER MARKETS: MANAGING THE WAIHOLE-WAIKANE AQUEDUCT

by

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Working Paper No. 00-6
April 2000

Constrained Conjunctive-Use for Endogenously Separable Water Markets: Managing the Waihole-Waikane Aqueduct*

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October 21, 1999

Abstract

Water districts relying on surface water can achieve overall efficiency by first attaining efficiency in their individual districts and then trading across districts. This is not the case when groundwater is an important source. Efficiency requires simultaneous planning for allocation within and across districts for current and future periods. Efficiency conditions are described and illustrated for two Oahu districts interlinked by a common source of water from the mountains between them. Centralized and decentralized mechanisms for implementing the efficient

*Keywords: resource economics, conjunctive-use

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solution are discussed.

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Abstract

An internal solution to an optimal control problem involving conjunctive use of surface and groundwater may be inapplicable if water is not sufficiently fungible across space and time. We provide a more general solution and apply it to the problem of allocating a limited amount of water from the Koolau mountains to two Oahu water districts separated by those mountains. The solution involves initially allocating all of the mountain water to the district supplied by groundwater but eventually allocating all of the water to the district supplied by surface water. The conditions for an internal solution hold only in the intervening years when some mountain water is allocated to each district.

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1. Introduction

As in many states, water management in Hawaii is organized according to separate water districts. It is commonly assumed that efficient management can be accomplished by managing each district separately and then trading across districts until the value of water is equalized across districts. This view not only glosses over spatial issues such as transport costs and conveyance losses, but also overlooks complications that arise in the context of conjunctive use. Previous authors have shown how to incorporate transportation costs and water quality into water allocation models such that the marginal benefit at each point in the system is equal to its corresponding full marginal cost, including the conveyance costs as well as pollution costs (see e.g. Chakravorty et. al, 1995; and McConnell and Strand, 1998). Similar conditions apply to the problem of interbasin water transfers. Such models, however, require that preconditions guarantee the feasibility of an internal solution. When it is possible to transport the resource from one market to the other, however, one could encounter situations where the amount of water available for transport is insufficient to equate water values across districts, and hence, preempt the possibility of obtaining internal solutions.

In the present paper, we are concerned with a case in which there are limits to allocating water between neighboring markets. For illustrative purposes we consider a situation in Hawaii involving two water districts that share a common source. Because each district has its own source as well as the common source, corner solutions are possible in which all of the common source is allocated to one district or the other. The situation is further complicated by the problem of conjunctive use; one district relies primarily on groundwater from the Pearl Harbor Aquifer, while the other relies primarily on surface water. Ordinarily, one would solve for the extraction/allocation profile that simultaneously solves for optimal depletion of the aquifer up to some steady state and optimal spatial distribution of the total water available in each time period (Tsur, 1991 and Tsur and Graham-Tomasi, 1991). In the present problem, however, one must solve for constrained conjunctive-use, admitting the possibility that water scarcity may become greater in one market than the other – even when it already receives all of the available common water. Since the optimal allocation may be different in each time period, this involves choosing an allocation vector for the common source as well as an optimal extraction profile for the groundwater aquifer, both of which are interdependent. The model we present is a step in the direction of developing a general spatial/intertemporal model of conjunctive use water management with multiple water sources and transport technologies.

Section 2 below provides a theory of conjunctive use with limited possibilities of moving water from one district to another. Section 3 illustrates the model for a current conflict between water districts on the Island of Oahu. Some conclusions and policy implications are offered in section 4.

2. Model

Consider two adjacent water districts divided by a mountain range, with districts 1 and 2 indexed by $i = 1, 2$. Each district contains a fully integrated water market with its own aggregate demand function. An aqueduct system traverses the mountain range and the constant flow of water through the aqueduct can be used to supply either district with water. Denote the daily flow of aqueduct water by $\bar{S}$ and the amount of aqueduct water diverted to districts 1 and 2 at time t by $s_1(t)$ and $s_2(t)$ respectively. The per unit cost of gravity driven transport of the water to district i is denoted $\tau_{0i} \geq 0$.[1]

[1] Each district's transportation cost may be thought of as the mean cost to that district. For methods of incorporating intra-district transport cost differences, see Chakravorty and Roumasset (1988) and Chakravorty et al.

District 1 has access to a coastal aquifer, where the amount of aquifer water extracted at time t is denoted $q(t)$. We follow Krulce et al in modeling aquifer characteristics. Let $h(t)$ denote the head of the aquifer above sea level at time t, and let $l(h)$ denote the amount of water leaking from the aquifer given head level h. The higher the head level, the larger the surface area from which water can leak, and the greater the water pressure on the existing surface area; suggesting that leakage increases in head. We assume that the leakage function satisfies the following properties: $l(h) \geq 0$, $l'(h) > 0$, $l''(h) \geq 0$, and $l(0) = 0$, i.e., leakage is a positive, increasing, convex function of head. Aquifer inflow (from rainwater) occurs at rate w. Unexploited, the aquifer head rises to the level $\bar{h}$ where leakage exactly equals inflow, $w = l(\bar{h})$. Since leakage increases as head levels increase and head levels fall as the aquifer is exploited, it follows that $w - l(h) \geq 0$. The aquifer head evolves over time according to $\dot{h}(t) = w - l(h(t)) - q(t)$.[2] The average cost of extracting water from the aquifer is $c(h) \geq 0$, where $c'(h) < 0, c''(h) \geq 0$, and $\lim_{h \to 0} c(h) = \infty$.

In addition to aqueduct water, district 2 receives a daily flow of surface water and sustainable groundwater yields denoted S^F. Both districts have access to exotic backstop technologies, for example desalination. Let $b_1(t)$ and $b_2(t)$ denote the time t amount of the backstop resource supplied to districts 1 and 2 respectively, and represent the per-unit cost of the backstop technology (desalination) by $\bar{p}$. The per unit cost of transporting aquifer or backstop water to end users in district 1 is denoted $\tau_1 \geq 0$, while the per unit cost of transporting surface or backstop water to end users in district 2 is denoted $\tau_2 \geq 0$.

Given S^F and access to the backstop technologies, a water commission is responsible for managing aqueduct allocation and water extraction rates from the aquifer. The time t water demands for districts 1 and 2 are represented by $D^1(p_1(t), t)$ and $D^2(p_2(t), t)$ respectively. Here $p_i(t)$ is the time t price of water in district i. For $i = 1, 2$ and for all t we assume $D_1^i = \partial D_i / \partial p_i < 0$

(1995).

[2] All dotted variables, e.g., $\dot{h}$, refer to time derivatives.

and $D_2^i = \partial D^i/\partial t \geq 0$: demand is strictly decreasing in price and demand is nondecreasing over time. Denote the time t inverse demands for district i by $N^i\left(x_t^i, t\right)$, where x_t^i is the time t quantity of water demanded in district i. Given the properties of D^i, it follows that $N_1^i = \partial N^i/\partial x_t^i < 0$ and $N_2^i = \partial N^i/\partial t \geq 0$, $i = 1, 2$. The gross surplus of water in district i is given by $\int N^i(x,t)\,dx$. Then the water commission's problem can be represented as choosing the trajectory $\{q(t), b_1(t), s_1(t), b_2(t), s_2(t)\}_{t\in(0,\infty)}$ to maximize:

$$\int_0^{\infty} e^{-rt}\left\{\int_0^{q+b_1+s_1} N^1(x,t)\,dx - \left[c(h(t))\,q(t) + \bar{p}b_1(t) + s_1(t)\,\tau_{01} + (q(t)+b_1(t))\,\tau_1\right]\right.$$
$$\left. + \int_0^{S^F+b_2+s_2} N^2(x,t)\,dx - \left[\bar{p}b_2(t) + s_2(t)\,\tau_{02} + \left(S^F + b_2(t)\right)\tau_2\right]\right\} dt \tag{2.1}$$

subject to:

$$\dot{h}(t) = w - l(h(t)) - q(t),$$

$$\bar{S} = s_1(t) + s_2(t),$$

$$0 \leq s_i(t) \leq \bar{S},\; i = 1, 2\;.$$

Using $s_2(t) = \bar{S} - s_1(t)$, the current value Hamiltonian for this problem is:

$$H = \int_0^{q_t+b_1+s_1} N^1(x,t)\,dx - \left[c(h(t))\,q(t) + \bar{p}b_1(t) + s_1(t)\,\tau_{01} + (q(t)+b_1(t))\,\tau_1\right] + s_1(t)\,\gamma_1(t)$$
$$+ \int_0^{S^F+\bar{S}-s_1+b_2} N^1(x,t)\,dx - \left[\bar{p}b_2(t) + \left(\bar{S} - s_1(t)\right)\tau_{02} + \left(S^F + b_2(t)\right)\tau_2\right] + \lambda(t)\left[w - l(h(t)) - q(t)\right]$$
$$+ \left[\bar{S} - s_1(t)\right]\gamma_2(t)\,.$$

Here $\lambda(t) \geq 0$ is the time t shadow price of aquifer water, and $\gamma_i(t) \geq 0$ is the time t shadow value of an additional unit of aqueduct water to district i. Suppressing t, the necessary conditions for an optimal solution are

$$\dot{h} = w - l(h) - q \tag{2.2}$$

$$\dot{\lambda} = r\lambda - \frac{\partial H}{\partial h} = r\lambda + c'(h)\,q + l'(h)\,\lambda \tag{2.3}$$

$$\frac{\partial H}{\partial q} = N^1(q + b_1 + s_1, \cdot) - c(h) - \tau_1 - \lambda \leq 0 \tag{2.4}$$

$$\frac{\partial H}{\partial b_1} = N^1(q + b_1 + s_1, \cdot) - \bar{p} - \tau_1 \leq 0 \tag{2.5}$$

$$\frac{\partial H}{\partial s_1} = N^1(q + b_1 + s_1, \cdot) - \tau_{01} + \gamma_1 - N^2\left(S^F + \bar{S} - s_1 + b_2, \cdot\right) + \tau_{02} - \gamma_2 \leq 0 \tag{2.6}$$

$$\frac{\partial H}{\partial b_2} = N^2\left(S^F + \bar{S} - s_1 + b_2, \cdot\right) - \bar{p} - \tau_2 \leq 0 \tag{2.7}$$

$$\frac{\partial H}{\partial \gamma_1} = s_1$$

$$\frac{\partial H}{\partial \gamma_2} = \bar{S} - s_1$$

and the complementary slackness conditions:

$$\frac{\partial H}{\partial q} q = \frac{\partial H}{\partial b_i} b_i = \frac{\partial H}{\partial s_i} s_i = \frac{\partial H}{\partial \gamma_i} \gamma_i = 0;\ i = 1, 2. \tag{2.8}$$

Define $p_1(t) \equiv N^1(q(t) + b_1(t) + s_1(t), t)$ and $p_2(t) \equiv N^2\left(S^F + \bar{S} - s_1 + b_2, t\right)$. Following Krulce et al, we assume the cost of desalination is high enough so that water is always extracted from the aquifer and (2.4) always holds with equality,

$$\lambda = p_1 - c(h) - \tau_1. \tag{2.9}$$

By equation (2.9), the *in situ* shadow price of district one water, λ, is equal to the market price of water in district 1 less extraction and transport costs. Also, rearranging equation (2.3) gives

$$\lambda = \frac{\dot{\lambda}}{r} - \frac{c'(h)\,q}{r} - \frac{l'(h)\,\lambda}{r}. \tag{2.10}$$

The left-hand side of (2.10) is the marginal benefit of extracting water today. The right-hand-side is the marginal user cost, i.e. the lost present value, of extracting water. The first right-hand-side term is the foregone present value from not appropriating the capital gain associated with saving the marginal unit of water. The second term is the present value lost from having to incur higher extraction costs in the future. The third term is the (partially offsetting) reduction in marginal user cost due to the higher recharge associated with the lower head level. Alternatively, expression (2.10) can be rewritten as

$$\lambda = \frac{\dot{\lambda} - c'(h)\, q}{r + l'(h)},$$

where the right hand side is an alternative form of marginal user cost and where the denominator can be thought of as the own interest rate associated with saving water.

Equations (2.5) and (2.7) describe what happens to market price when the backstop technology is adopted. These equations can be rewritten as

$$p_i - \tau_i \leq \bar{p}, \quad i = 1, 2. \tag{2.11}$$

If the shadow price of water in district i is less than the cost of desalination, then desalination does not occur on that side. When either district uses desalination, the price of water on that side must be equal to the per-unit cost of desalination, i.e., $p_i - \tau_i = \bar{p}$. Hence with desalination, $p_i - \tau_i = \bar{p}$ implies that expression (2.3) can be written $\lambda = \bar{p} - c(h)$; the in situ shadow price of water varies only with average extraction costs, i.e., $\dot{\lambda} = -c'(h)\, \dot{h}$. Next, combine $\lambda = \bar{p} - c(h) - \tau_1$ and $\dot{\lambda} = c'(h)\, \dot{h}$ with equations (2.2) and (2.3), and eliminate $\lambda, \dot{\lambda}$, and $\dot{h}$. Using we see that when desalination is adopted in district 1

$$\bar{p} - c(h) = -\frac{[w - l(h)]\, c'(h)}{r + l'(h)} > 0. \tag{2.12}$$

Recall from (2.9) that the left hand side of equation (2.12) is the in situ shadow price of water, i.e. the marginal benefit of extracting water in the optimal program. The right hand side is the marginal user cost once the aquifer reaches a steady state. Note that, unlike the case of non-renewable resources, the scarcity rent, λ, does not go to zero when the backstop is employed. As shown in the application, the steady state scarcity rent can, in fact, be several times larger than the extraction cost.

Krulce et al argue that the optimal head level satisfying (2.3) is unique and when desalination is used the optimal head level is maintained at a constant level, denoted h^*. In such a case, extraction rates must be equal to the net inflow of water to the aquifer and extraction rates are constant, denoted q^*.

2.1. Aqueduct Management

The optimal allocation of aqueduct water is governed by equation (2.6) and must satisfy the following conditions:

$$p_1(t) - \tau_1 = p_2(t) - \tau_2, \qquad \gamma_1(t) = \gamma_2(t) = 0,$$

$$p_1(t) - \tau_1 > p_2(t) - \tau_2, \qquad \gamma_1(t) = 0,\ \gamma_2(t) > 0,$$

$$p_1(t) - \tau_1 < p_2(t) - \tau_2, \qquad \gamma_1(t) > 0,\ \gamma_2(t) = 0.$$

If the equilibrium price paths are moving together then both sides are receiving aqueduct water. If over a period of time equilibrium prices are such that $p_1(t) - \tau_1 < p_2(t) - \tau_2$, then at some earlier point in time district 2 received (and continues to receive) all of the aqueduct water $(s_{1t} = 0)$. If, instead $p_1(t) - \tau_1 > p_2(t) - \tau_2$, then at some earlier point in time district 1 received all of the aqueduct water $(s_1 = \bar{S})$. As one might imagine, the higher the transport cost is for one district relative to the other, the less aqueduct water the relatively higher transport cost district will receive.

When prices have diverged the allocation rule is simple: divert all of the aqueduct water to the side with the highest price. However, when prices are moving together and the backstop technology has not been adopted, the allocation rule is a bit more complex. To see how s_1 behaves in such a case, take the time derivative of the equilibrium relationship (2.6):

$$\frac{d}{dt}\left(\frac{\partial H}{\partial s_1}\right) = \frac{d}{dt}\left[N^1\left(q + b_1 + s_1, t\right) - N^2\left(S^F + b_2 + \bar{S} - s_1, t\right)\right]$$
$$= \left(\dot{q} + \dot{b}_1 + \dot{s}_1\right) N_1^1\left(\cdot,\cdot\right) + N_2^1\left(\cdot,\cdot\right) - \left(\dot{b}_2 - \dot{s}_1\right) N_1^2\left(\cdot,\cdot\right) - N_2^2\left(\cdot,\cdot\right). \tag{2.13}$$

Given that prices are moving together it follows that $\frac{d}{dt}\left(\frac{\partial H}{\partial s_1}\right) = 0$. Also, since the backstop technology has not been reached: $\dot{b}_1 = \dot{b}_2 = 0$. Then,

$$N_2^1 + \left(\dot{q} + \dot{s}_1\right) N_1^1 = N_2^2 - \dot{s}_1 N_1^2. \tag{2.14}$$

Price movements in each market are the result of two effects. Holding water consumption constant, the time effect N_2^i is the change in price induced by a shift in district i demand at time t. The direct demand effects $\left(\dot{q} + \dot{s}_1\right) N_1^1$ and $-\dot{s}_1 N_1^2$ are the respective price responses in districts 1 and 2 to changes in consumption levels at time t. For either market, if the time effect dominates the direct demand effects, then prices increase. Otherwise, prices remain constant or fall.

In equilibrium, quantity demanded must equal quantity supplied, or $q + s_1 = D^1\left(p_1, t\right)$. Taking the time derivative of this equilibrium condition gives $\dot{q} + \dot{s}_1 = D_1^1 \dot{p}_1 + D_2^1$. Substituting this time derivative into expression (2.14) and rearranging terms gives the desired behavior of s_1:

$$\dot{s}_1 = \frac{N_2^2 - \left[N_2^1 + \left(D_1^1 \dot{p}_1 + D_2^1\right) N_1^1\right]}{N_1^2}. \tag{2.15}$$

Given that $N_1^2 < 0$, if the second district's shift in demand is large (small) enough relative to the total changes in the other district's demand, then the amount of aqueduct water diverted to district 2 should be increased (decreased). There are potentially many patterns of optimal aqueduct water diversion schemes. For example, if prices are increasing and demand in district 2 is always increasing more quickly over time than demand in district 1, then more and more aqueduct water will be shifted to district 2. However, even if prices are increasing and demand in district 1 is shifting more rapidly than that in district 2, it is unclear whether district 1 should receive an increasing share of aqueduct water. For instance, if the first district's direct demand effects are large enough relative to its time effects, then it is possible for the time shifts in the second district to dominate the first district's net demand effects. In such a case the first district would receive increasing quantities of aqueduct water.

If prices move together and reach the backstop at the same time, then the amount of water to divert to either side is arbitrary – the desalination costs saved by the aqueduct water is the same regardless of its allocation. If backstop technology costs were higher on one side than the other, however, then the water manager is not indifferent about where the aqueduct water is sent. One would expect that if desalination were higher in district 2 than district 1, then if district 1 reached the backstop technology first then district 1 could possibly receive all of the aqueduct water. Then after the price in district 2 reached the backstop price in district 1, district 2 would eventually receive all of the aqueduct water and keep it even after the it reached the backstop technology.

2.2. The Optimal Price and Quantity Trajectories

The choice of q, s_1, b_1 and b_2 must satisfy several conditions simultaneously. To facilitate the algorithm design we observe that equation (2.6) can be used to define s_1 in terms of q, b_1, and b_2. The relationship between these variable is captured in the following *aqueduct response function*:

$$\begin{aligned} s_1^*(q, b_1, b_2) &= \arg\min_{s_1} \left\{ \left| N^1 (q + b_1 + s_1, \cdot) - \tau_{01} - \left[N^2 \left(S^F + b_2 + \bar{S} - s_1, \cdot \right) + \tau_{02} \right] \right| \right\} \quad (2.16) \\ \text{subject to } s_1 &\in \left[0, \bar{S}\right], \quad q, b_1, b_2 \geq 0, \end{aligned}$$

where the aqueduct response function gives the optimal rate at which aqueduct water should be diverted to district 1, given desalination and aquifer extraction rates. Introducing the aqueduct response function into the optimal control problem is accomplished by noting that, in equilibrium, water supply in district 1 must be equal to its quantity demanded:, i.e., $q + b_1 + s_1^* = D^1(p_1, \cdot)$, or

$$q = D^1(p_1, \cdot) - b_1 - s_1^*(q, b_1, b_2). \tag{2.17}$$

Then, let q^* denote the (possibly implicit) solution to (2.17), and substitute q^* into (2.2). Finally, combining $\lambda = \lambda = p_1 - c(h) - \tau_1$ and $\dot{\lambda} = \dot{p}_1 - c'(h)\dot{h}$ with equations (2.2) and (2.3) [and eliminating $\lambda, \dot{\lambda}$, and $\dot{h}$] yields the following system of differential equations

$$\dot{h}(t) = w - l(h(t)) - q^*(t), \tag{2.18}$$

$$\dot{p}_1(t) = \left[r + l'(h(t))\right][p_1(t) - c(h(t)) - \tau_1] + [w - l(h(t))]\,c'(h(t)). \tag{2.19}$$

Equation (2.18) describes the optimal trajectory of the aquifer head and equation (2.19) is the optimal trajectory of district 1 water price. The optimal trajectory of s_1^* is recovered using q^*, and the optimal trajectory of district 2 water price is recovered by substituting s_1^* into that district's inverse demand curve.

In principle, several types of price trajectories are possible. For instance, both price trajectories might be such that desalination is never warranted. This might happen if demand in both districts grew slowly and leveled off before prices rose above the cost of desalination. However, if demand in one of the districts was high enough or grew fast enough, then desalination eventually will be adopted in that market. If the price trajectories were such that one district adopted desalination before the other, then necessarily, at some earlier point in time that district would have received all of the aqueduct water. For example, say the relative acceleration of water demand is higher in the second district. In such a case, in order to equate prices across markets the second district would receive more and more aqueduct water. Eventually the second district would get all of the aqueduct water, after which the second district's price would begin rising more rapidly than water prices in the first district. Once the price in district 2 reached backstop levels, desalination would begin on that side. As for the price in district 1, it would eventually either reach backstop levels or level off at some level below district 2 levels (possibly even fall). Several alternative scenarios are examined in the following section based on a situation in Oahu in the State of Hawaii.

3. Application: Optimal water management on Oahu

To illustrate an application of the above principles we reconsider the water management problem investigated by Moncur et al, where the relative merits of diverting a constant amount of surface water to each side was investigated. However, as the authors suggested, the optimal allocation of aqueduct water would likely vary over time. A numerical analysis of the interbasin transfer problem requires specification of the leakage function $l(h)$, the extraction cost function $c(h)$, and the inverse demand functions for the leeward side (district 1) and the windward side (district 2), $N^1(\cdot,\cdot)$ and $N^2(\cdot,\cdot)$.

Using the hydrological studies of Mink, the leakage function estimated by Krulce et al is $l(h) = 0.2497h^2 + 0.022h$, where $l(h)$ is measured in mgd (million gallons per day) and $h \in (0, 33.5)$. The extraction cost function used in Krulce et al is $c(h) = c_0\left(\frac{h_0}{h}\right)^n$ where $c_0 = \$0.283$ is initial extraction costs in 1991, $h_0 = 15$ is the initial head in 1991, and $n = 2$ is the rate at which extraction costs approach infinity (see Moncur and Pollock).

Both windward and leeward water demand comes from agricultural and urban sources. However, current projections suggest leeward urban water demand will grow over the foreseeable future, while leeward agricultural water demand is expected to fall. Accordingly, we decompose leeward demand into residential and agricultural demand. We assume windward urban and agricultural demand will grow at the same rate. We represent leeward demand by $D_1(p_{1t}, t) = \alpha_{11}e^{g_{11}t}(p_{1t} + c_{D_1})^{-\eta} + \alpha_{12}e^{g_{12}t}(p_{1t} + c_{D_1})^{-\eta}$, where $\alpha_{11}e^{g_{11}t}(p_{1t} + c_{D_1})^{-\eta}$ is leeward urban demand. Windward demand is represented by $D_2(p_{2t}, t) = \alpha_2 e^{g_2 t}(p_{2t} + c_{D_2})^{-\eta}$from $i = 1, 2$. Here p_{it} is the time t wholesale price of water in market i, c_{D_i} is distribution cost (net transport costs) in district i, η is the elasticity of demand, g_{11}, g_{12}, and g_2 are the growth rates of leeward urban, leeward agricultural, and windward demand respectively. The parameters α_{11} and α_{12} normalize leeward urban and agricultural demand to actual 1991 price and quantity data, while α_2 normalizes windward demand. Following Moncur et al, we set $c_{D_i} = 0.597$ and $\eta = 0.3$, and calibrate $\alpha_{11} = 93.63$, $\alpha_{12} = 107.47$, and $\alpha_2 = 40.43$ (see Krulce et al, pg. 1223). Per-unit transport costs are assumed to be given by $\tau_1 = 0.25$ and $\tau_2 = 0.45$. To derive α_2 we observe that 1991 windward water demand is 38 mgd, implying $\alpha_2 = 38 * 1.23^{0.3} = 40.43$. Finally we assume $S^F = 38000$ mgd, $\bar{S} = 28000$ mgd, $\bar{p} = 3$[3] and the real interest rate r is equal to 3% (see Roumasset, Isaak, and Fesharaki). Relatively straightforward manipulations yield

$$p_{1t} = N^1\left(q + b_1 + s_1, t\right) = \left(\frac{93.63\ e^{g_{11}t} + 107.47\ e^{g_{12}t}}{q + b_1 + s_1}\right)^{\frac{1}{0.3}} - 0.847,$$

$$p_{2t} = N^2\left(S^F + \bar{S} + b_2 - s_1, t\right) = \left(\frac{40.43\ e^{g_2 t}}{S^F + \bar{S} + b_2 - s_1}\right)^{\frac{1}{0.3}} - 1.047.$$

In the following analysis we consider three scenarios. In two of the scenarios leeward demand grows at 2% each year, and windward demand grows at either 1.5% or 0.4% each year. In the last scenario leeward demand grows at 1.4% and windward demand grows at 0.5% each year. In each scenario leeward agriculture is assumed to grow at -1%.

In the first scenario urban water demand on the leeward side grows at 2% while windward demand grows at 1.5%. The price and aqueduct water management trajectories corresponding to this scenario are presented in figures 1 and 2. In figure 1 the optimal price trajectory for the windward side is given by the dashed curve, while the optimal price trajectory for the leeward side is given by the heavy, shaded curve. Note that the leeward price lies above the windward price until a little after 2002, after which the prices move together until a little after 2047. Then the windward price increases more rapidly than the leeward price, hitting the backstop technology about two or

[3] See Leitner. While Leitner's estimate is in 1984 dollars and ours is in 1991 dollars we assume technilogical change just offsets inflation in the intervening periods.

three years later. Figure 2 shows that the optimal aqueduct water diversion pattern corresponds with what one might expect. In the first few years the leeward side gets all of the aqueduct water. However, a little after 2002 the windward side begins receiving some of the aqueduct water, and continues to receive increasing amounts until, in about 2047, it gets all available aqueduct water. This happens because the leeward side can meet increased demand needs by drawing more and more water out of its coastal aquifer, while the windward side faces a relatively fixed source of supply, and hence in order to meet future increased water demand, must resort to the aqueduct water. Without the benefit of the aqueduct source, the value of water would correspondingly rise more rapidly on the windward side. Accordingly, the windward side commands an increasingly larger share of the aqueduct water in order to keep the marginal value of aqueduct water equal across districts. The equality breaks down after the corner solution is reached with all water going to the windward side.

Figure 3 corresponds to the case where leeward urban and windward demands grow at 2% and 0.2% per year respectively. In this case windward demand grows so slowly relative to leeward demand that the windward water price never rises above the leeward water price. Correspondingly the windward side never receives any aqueduct water.

Finally, figures 4 and 5 correspond to the case where urban leeward demand grows at 1.4% each year while windward demand grows at 0.5% each year. This example is presented to show that monotonic diversion patterns are not necessarily the rule. In figure 4 the optimal price trajectory on the leeward side is the heavy, shaded curve, while the optimal price trajectory for the windward side is the dashed curve. From 1991 until about 1997 leeward prices are higher than windward prices. After which the price trajectories are identical and both prices reach the backstop technology in 2095. In figure 5, we see that it is optimal to allocate the leeward side all of the aqueduct water until about 1997. Then the windward side should begin receiving a monotonically increasingly share of the tunnel water until about 2052, after which the windward allocation should monotonically decrease until 2083. Finally, the windward side should again receive increasing shares of the tunnel water until the backstop technology is reached in 2095.

As with the analytical results presented in section 2, the illustrative exercises presented here suggest that the optimal extraction vector and aqueduct sharing rules are interdependent and, hence the need to coordinate aquifer extraction rates and aqueduct sharing rules. Failure to do so will inevitably lead to inefficient water allocations. As a final note, observe that assigning water rights and allowing water trading would tend to ameliorate inefficiencies, but, even aside from externalities, full efficiency would require that future markets exist for several decades into the future.

4. Conclusion

The model presented here is a step in the direction of developing a general spatial/intertemporal model of conjunctive-use water management with multiple water sources and transport technologies. The usual assumptions that water sources are at a single location or that transport possibilities are characterizable by a matrix of transport coefficients (linear transportation costs) are highly restrictive and typically misrepresent inter-basin transfer possibilities. The procedure we outline requires solving simultaneously for production at each source and time, and consumption in each district in each time period. The method can be generalized further to distinguish different locations within districts and to determine flow rates in each part of the conveyance system at each time.

The model focuses on two water districts, each with their own sources, but with each having potential access to a common, albeit limited, source. The optimal solution involves allocating each successive unit of common water to the district with the highest marginal water value. If water is sufficient, this will lead to equalization of marginal values across districts. If not, the entire amount of water will be allocated to one district or the other, and the efficiency prices will diverge across districts. In general, it is not possible to determine a priori whether the districts will be integrated in the sense of having the same efficiency prices or will be analytically separate.

If both districts rely on surface water, then optimal allocation of each district can be solved separately in each period and the common water allocated as described above in each period. This method is not available if one or both of the districts relies in part on groundwater. In that case, the periods are interdependent. One must solve simultaneously for the optimal path of groundwater extraction and the optimal allocation path of the common water.

The Hawaii application provides a number of lessons that may be of general interest. To the extent that groundwater is underpriced to a greater extent than surface water, as in the two Oahu districts, then the groundwater district should receive initial priority in allocating water from the common source. However, this assumes that the water authorities will simultaneously adopt efficiency pricing or some other mechanism for efficient water allocation. There is no point in allocating more water to the district where water is scarce if that district will continue to waste it.

But while water may be initially scarcer in the groundwater district, the situation may be rapidly overturned if new water sources are not available in the surface-water district. The intertemporal fungibility of groundwater makes it possible to conserve water now as a device to moderate the otherwise scarcity-increasing effects of demand growth. Such an option is not necessarily available in the case of surface water, unless storage facilities are developed or some potential water sources are left undeveloped until some future date. Without such facilities, demand growth will eventually cause scarcity in the surface-water district to overtake that of the groundwater district, and priority for common-water allocation will switch from the groundwater district to the surface-water district, as in the Hawaii case.

A third lesson, derivative of the first two, is that reliable benefit-cost studies of investments in new water facilities (e.g., pumping stations, dams, aqueducts, tanks, and conveyance structures) cannot be performed without discovering the time-dependent scarcity value of water, thus requiring analogous simulations to those reported here.

The theory and the illustrative exercises underscore the inevitable inefficiency of attempting to manage two such water districts independently. One cannot, as in the Hawaii case for example, choose some initial allocation of the shared source and then attempt to adjust the allocation over time according to criteria of relative scarcity in the two districts. The initial allocation itself effects the dynamic path of efficiency prices, and relative scarcity changes over time. Water trading between the two districts would tend to ameliorate inefficiencies, but, even aside from externalities, full efficiency would require future markets for several decades ahead.

The algorithm discussed in section three renders the intertemporal optimal apportionment problem tractable. Other extensions that would improve the normative value of the model for generating recommendations about optimal management within and across districts include explicit recognition of differential conveyance costs as well as instream benefits and other external effects.

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Figures

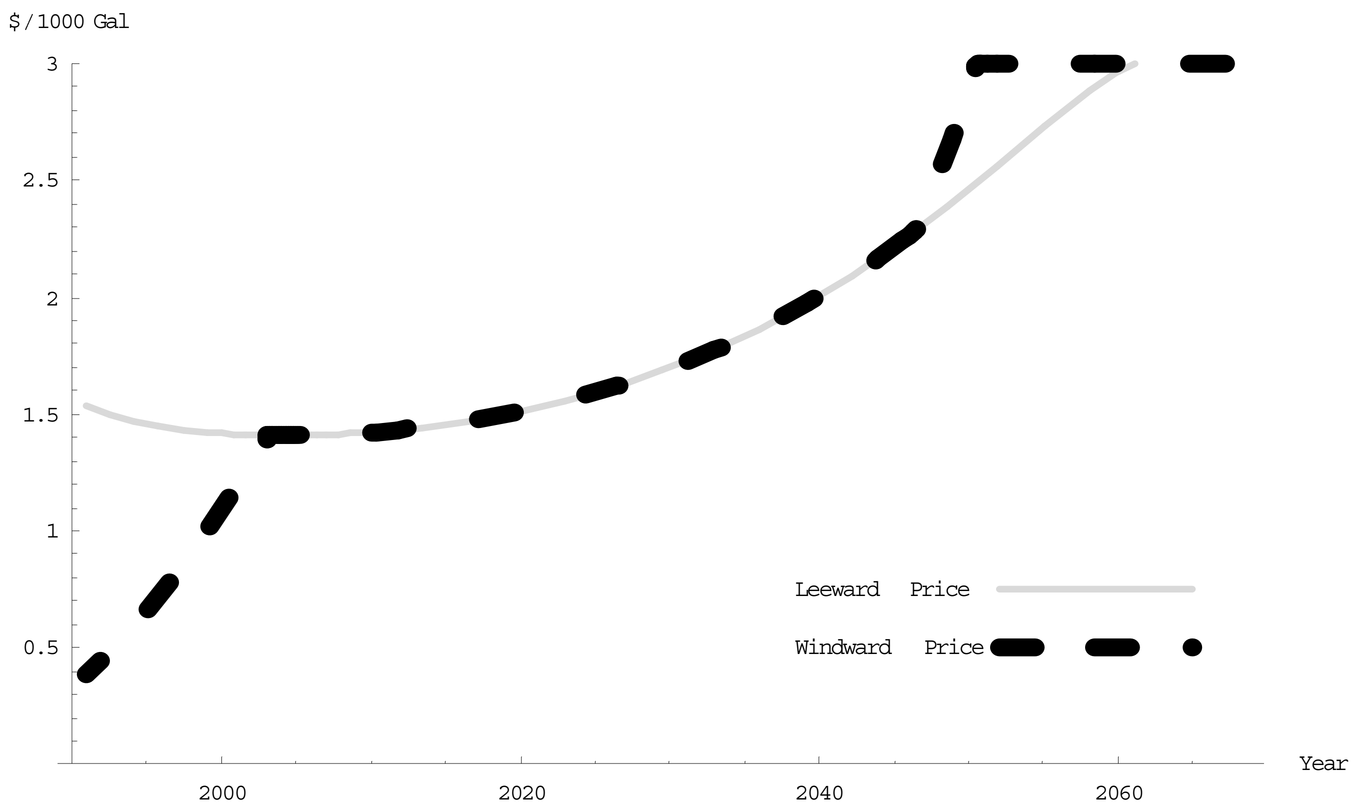

Figure 1. Optimal price trajectories when $g_1 = 2\%$, $g_2 = 1.5\%$.

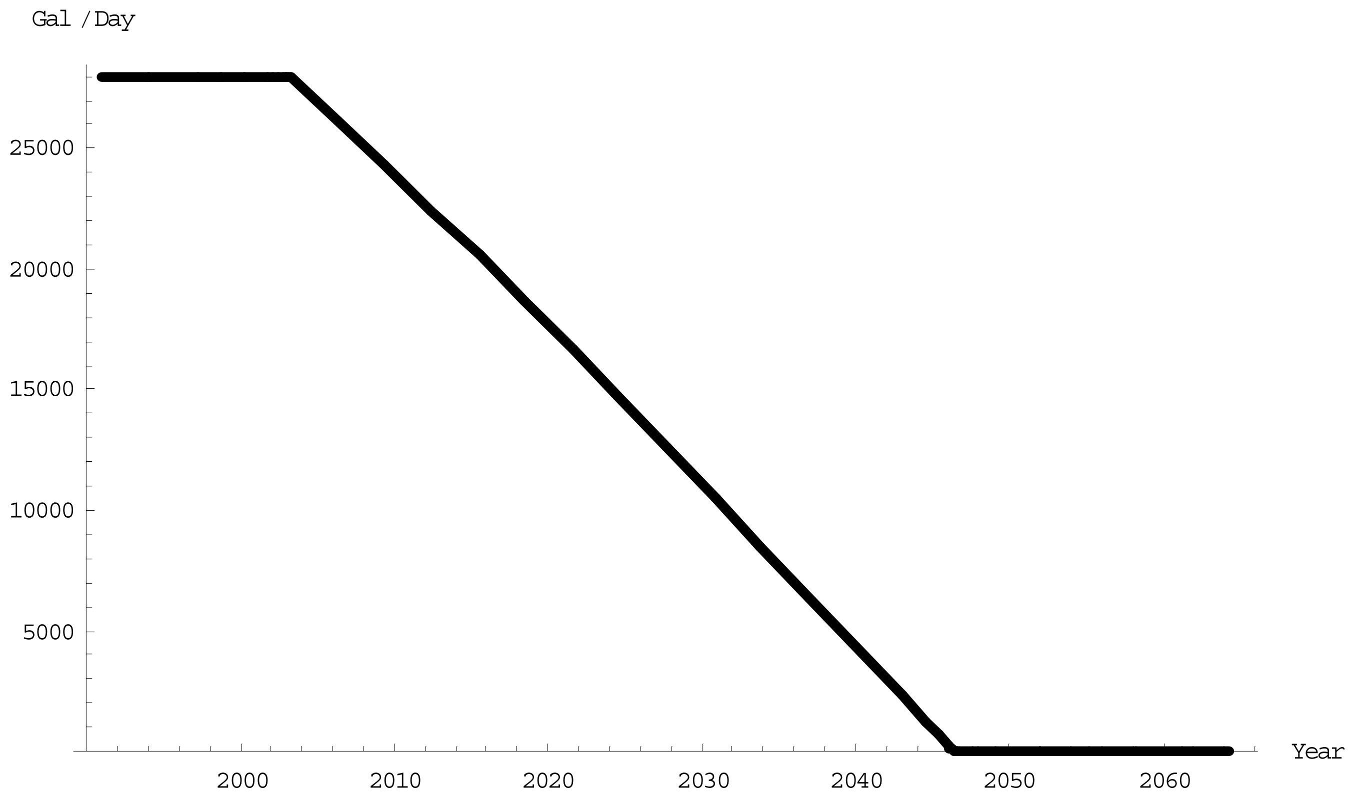

Figure 2. Optimal allocation of tunnel water to the Leeward market when $g_1 = 2\%$, $g_2 = 1.5\%$.

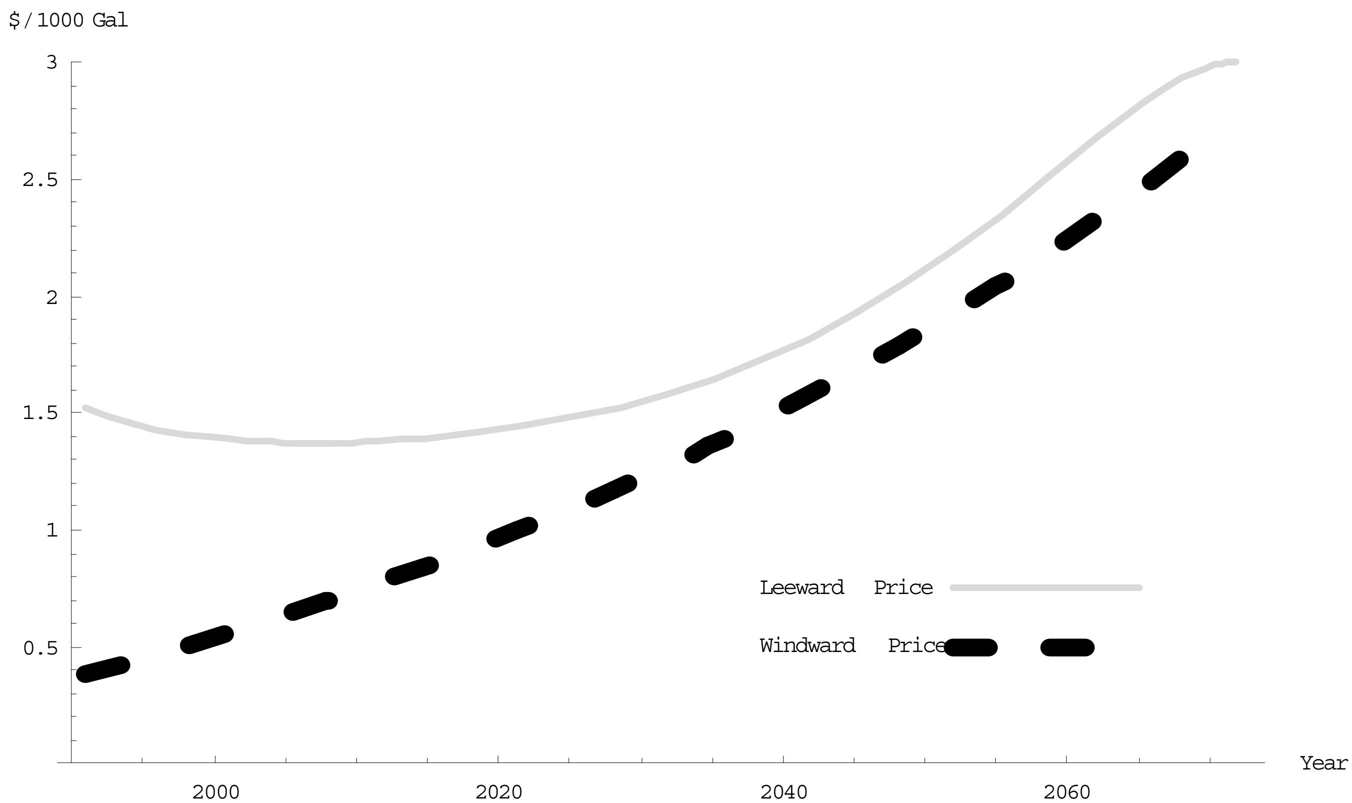

Figure 3. Optimal price trajectories when $g_1 = 2\%$ and $g_2 = 0.4\%$.

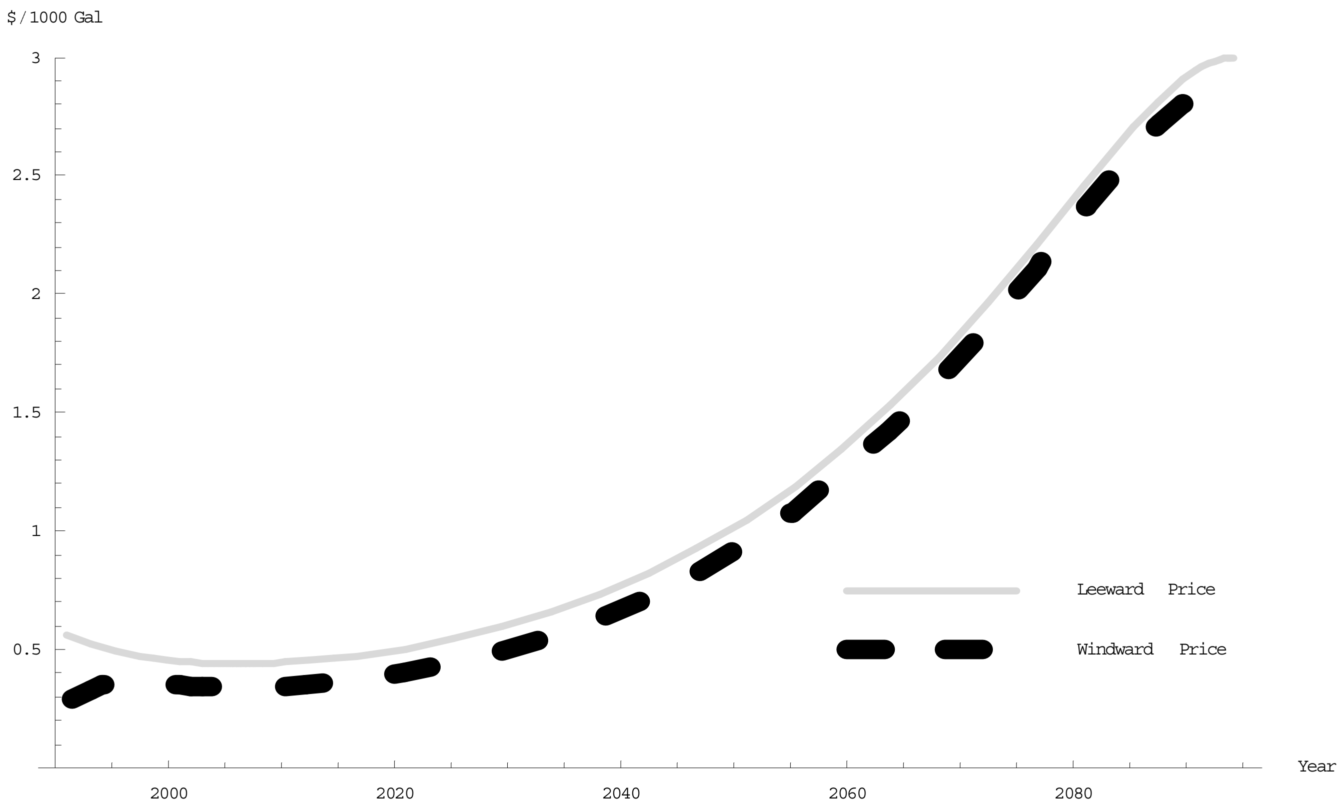

Figure 4. Optimal price trajectories when $g_1 = 1.4\%$ and $g_2 = 0.5\%$.

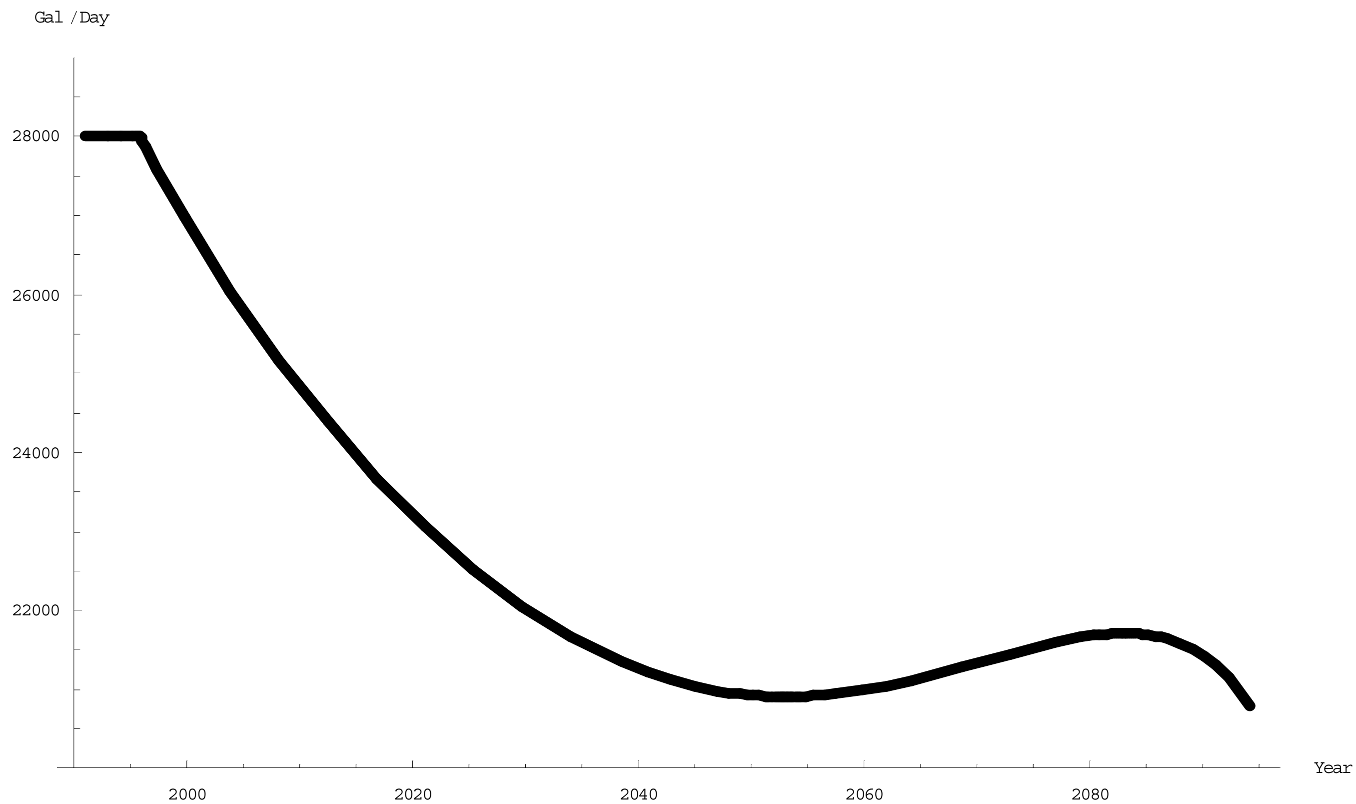

Figure 5. Reswitching in optimal sharing rules when $g_1 = 1.4\%$, $g_2 = 0.5\%$.