University of Hawai`i at Mānoa Department of Economics Working Paper Series

Saunders Hall 542, 2424 Maile Way,
Honolulu, HI 96822
Phone: (808) 956 -8496
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Working Paper No. 21-06

On the Robustness of Pricing Mechanisms

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December 2021

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December 24, 2021

Abstract

We study the robustness of two well-known and frequently observed multilateral trading protocols, price posting and auction, in small markets. In the context of directed search, sellers choose and commit to an ex-ante trading protocol to attract buyers. When constructing equilibrium, the deviating seller usually chooses the same mechanism as non-deviating sellers. In this paper, however, we allow the deviating seller to bargain with a buyer. In this setup, we find that price posting and auction are not robust mechanisms, because there is always a profitable deviation of bargaining. Then, we introduce a new hybrid multilateral trading protocol, which combines auction and bargaining. We show that when sellers commit to such a mechanism, the equilibrium is robust to deviations of bargaining.

JEL Classification: D40, D83, E40.
Keywords: Auction, Bargaining, Directed Search, Price Posting.

*We thank Pedro Gomis-Porqueras for valuable comments. Han thanks the Philosophy and Social Sciences Major Project Fund of the Ministry of Education of China, No. 18JZD029 and School of Economics, Peking University for research support. Julien and Wang acknowledge the financial support from the ARC grant DP1701014229.

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1 Introduction

Search frictions are prevalent in many markets, yielding unemployment, unsold goods, singles looking for a marriage partner, among other phenomena. These are examples of the pervasive failure of the law of one price. To address this issue, the literature has considered environments where individuals choose what terms of trade to search for and consider the trade-off between the terms of trade and trading probability.[1] Over the last three decades, the directed search models have been fruitfully used in a variety of applications in areas such as labor, industrial organization, and monetary economics.[2]

This framework considers capacity-constrained sellers that first post and commit to a particular term of trade to attract uncoordinated buyers through costless, public, and perfect signals. After observing the terms of trade, uncoordinated buyers randomize over which seller(s) to visit. This is the case as a single seller cannot supply the entire market.[3] In this environment, the literature has primarily focused on two most prevalent trading protocols: (i) posting prices as in Burdett, Shi, and Wright (2001, BSW hereafter) and (ii) posting reservation values to use competing auctions as in Julien, Kennes and King (2000, JKK hereafter). These two mechanisms deliver a unique mixed-strategy symmetric equilibrium which is robust to a deviating seller that sticks to the same mechanism as non-deviating sellers but chooses a different terms of trade.[4] Some natural questions arise in such a framework. Are price posting and auction robust in situations where the deviating seller chooses a different pricing mechanism, such as bargaining, an ex-post mechanism? If these mechanisms are robust, which yields a larger payoff to sellers? The purpose of this paper is to answer these questions.

[1]Earlier literature considered environments where individuals know the terms of trade only after the match and bargain over the total surplus, as in Diamond (1982), Mortensen (1987), and Pissarides (2017). Alternatively, there is ex-ante price posting but the terms of trade do not influence who meets who, as in Burdett and Judd (1983) and Burdett and Mortensen (1998). In these settings, the meetings are exogenous.

[2]We refer the reader to Wright et al. (2021) for a comprehensive survey on directed search that endogenizes how agents meet.

[3]If multiple buyers approach the same seller, each buyer is served with equal probability.

[4]In the context of price posting, a single seller deviates and posts a different price, while in the context of auction, a single buyer deviates and posts another reservation value.

In the literature, it is an important and long-standing issue to understand what drives the choice of trading protocol. In large markets with informational asymmetries, McAfee (1993) argues that sellers choose auction in equilibrium when buyers are aware of the mechanism chosen by sellers before making contact. In contrast, Peters (1994) shows, in a similar environment, that sellers would choose to post prices if discount factors are large enough and buyers are unaware of the pricing mechanisms before they approach sellers. What would happen if there are no information asymmetries? In large markets with homogeneous buyers and sellers, Kultti (1999) and Julien et al. (2001) show that the expected payoff of sellers with price posting and auction are identical.[5] However, in small markets with a finite size, Julien et al. (2001) find that sellers' expected payoffs are higher if all sellers choose auction. Moreover, the difference in payoffs declines monotonically with market size. Similarly, Julien et al. (2002) show that in the 2×2 case, when sellers can sequence their decisions, i.e., choosing a pricing mechanism before choosing a specific price, both sellers auctioning with a reserve price then becomes the dominant strategy. In contrast to the previous literature, we study the robustness of price posting and auction when the deviating seller chooses a bilateral pricing mechanism, such as bargaining.

To do so, we consider an environment with a small market and directed search, where buyers and sellers do not face any asymmetric information, as in Julien et al. (2001). Sellers are able to commit ex-ante to the terms of trade, which are sent to all uncoordinated buyers costlessly. In particular, we consider price posting and auction, and allow for the deviating seller to bargain with a buyer. In this environment, we study the resulting mixed strategy equilibrium under the two trading protocols. We find that both price posting and auction are not robust to a deviation of bargaining. This is the case as there is always a profitable deviation to choose bargaining. Next, we also propose a multilateral trading protocol, an *auction-bargaining* mechanism, which is robust to a deviating seller who chooses bargaining. This hybrid trading protocol allows the seller to conduct an auction when multiple buyers

[5] Gomis-Porqueras et al. (2017) relax the perfect signal assumption and consider imperfect observability and costly informative advertising. They show that sellers would prefer price posting rather than auction.

visit him and bargain otherwise. Note that this type of mechanism is likely to provide the highest ex-ante surplus to sellers, compared to price posting and auction. This is because, if more than one buyer visits a seller, the seller gets all the surplus from trade by running an auction, and if only one buyer shows up, the seller obtains a share of the total surplus by bargaining with the buyer. We find that this hybrid mechanism is robust to bilateral deviations.

2 Mechanisms

Consider homogeneous buyers ($N \geq 2$) and sellers ($M \geq 2$). Each seller produces one unit of indivisible goods, which buyers value at unit utility. Following McAfee (1993), we study mechanisms, $\mu_i = \{p_n, \varepsilon_n\}_{n=1}^{N}$, that are characterized by a price p_n and a probability of trade ε_n, where n is the realized number of buyers meeting a seller. We restrict our attention to the following set of mechanisms.

$$
\begin{array}{ll}
\text{Price Posting:} & \mu_p = \left\{p_n = p, \varepsilon_n = \frac{1}{n}\right\} \\
\text{Auction:} & \mu_a = \left\{ \begin{array}{l} p_n = r, \varepsilon_n = 1 \text{ if } n = 1 \\ p_n = 1, \varepsilon_n = \frac{1}{n} \text{ if } n > 1 \end{array} \right\} \\
\text{Bargaining:} & \mu_b = \left\{p_n = \eta, \varepsilon_n = \frac{1}{n}\right\}
\end{array}
$$

where r is the seller's reservation price, and the buyer's highest bid is equal to unit utility. Under bargaining, sellers commit to a price of η, the seller's bargaining power, and a uniform trading probability. In multilateral meetings, one buyer is chosen at random to match with the seller and the rest of buyers cannot trade.

3 Equilibrium

In the following analysis, we focus on symmetric equilibrium where homogeneous buyers and sellers use the same strategy. Let $\Pi_i(\mu_i; \bar{\sigma}_i)$ and $U_i(\mu_i; \bar{\sigma}_i)$ be expected payoffs of sellers and buyers, respectively, where all sellers offer the same pricing mechanism μ_i and $\bar{\sigma}_i = (\sigma_i^1, ..., \sigma_i^M)$ represents the probabilities with which a buyer visits M different sellers, i.e., each buyer visits seller m with probability σ_i^m.

Assume one seller deviates to μ_j. Without loss of generality, we assume seller M deviates, and the rest of sellers stay with the original mechanism μ_i. Let $\Pi_j(\mu_j, \mu_i; \sigma_j, \bar{\sigma}_i')$ and $U_j(\mu_j, \mu_i; \sigma_j, \bar{\sigma}_i')$ be the payoffs of sellers and buyers if buyers choose the deviating seller, and $U_i(\mu_j, \mu_i; \sigma_j, \bar{\sigma}_i')$ represents buyers' payoff when they stay with non-deviating sellers. Then, σ_j is the probability with which buyers visit the deviating seller, and $\bar{\sigma}_i' = (\sigma_i^1, ..., \sigma_i^{M-1})$ represents the probabilities that buyers choose non-deviating sellers. The following definition characterizes the equilibrium mechanism.

Definition 1 *In any finite market, an equilibrium mechanism μ_i satisfies $\Pi_i(\mu_i; \bar{\sigma}_i) \geq \Pi_j(\mu_j, \mu_i; \sigma_j, \bar{\sigma}_i')$, i.e., the seller's payoff condition (SPC) and, $U_j(\mu_j, \mu_i; \sigma_j, \bar{\sigma}_i') = U_i(\mu_j, \mu_i; \sigma_j, \bar{\sigma}_i')$, i.e., the buyer's indifference condition (BIC) for all j, and $\sigma_j + \sum_{m=1}^{M-1} \sigma_i^m = 1$.*

Next, we derive conditions for an equilibrium to exist when one seller deviates to another price or mechanism. Note that in small markets the BIC changes when a seller deviates.

3.1 Price Posting

We first derive the price posting equilibrium when only one seller deviates to another price, which is discussed in BSW. Assume that all sellers but one use the same mechanism μ_p, where a candidate equilibrium price p is posted. Then, a seller deviates to mechanism $\mu_{\tilde{p}}$ where price $\tilde{p} \neq p$ is offered. Let $\sigma_{\tilde{p}}$ be the probability with which buyers visit the deviating seller, and $\bar{\sigma}_p' = (\sigma_p^1, ..., \sigma_p^{M-1})$ represents the probabilities that buyers choose sellers who do not deviate, where $\sigma_p^m = \sigma_p$, $\forall m \in \{1, ..., M-1\}$. Now the seller's payoff function $\Pi_{\tilde{p}}(\mu_{\tilde{p}}, \mu_p; \sigma_{\tilde{p}}, \bar{\sigma}_p')$ can be simplified to $\Pi_{\tilde{p}}(\mu_{\tilde{p}}, \mu_p; \sigma_{\tilde{p}}, \sigma_p)$, and the buyer's value function $U_{\tilde{p}}(\mu_{\tilde{p}}, \mu_p; \sigma_{\tilde{p}}, \bar{\sigma}_p')$ becomes $U_{\tilde{p}}(\mu_{\tilde{p}}, \mu_p; \sigma_{\tilde{p}}, \sigma_p)$.

The equilibrium outcome is derived as follows. The deviating seller chooses $\tilde{p}$ to maximize his profit

$$\Pi_{\widetilde{p}}(\mu_{\widetilde{p}}, \mu_p; \sigma_{\widetilde{p}}, \sigma_p) = \max_{\widetilde{p}}[1-(1-\sigma_{\widetilde{p}})^N]\widetilde{p},$$

subject to the BIC

$$U_{\widetilde{p}}(\mu_{\widetilde{p}}, \mu_p; \sigma_{\widetilde{p}}, \sigma_p) = \frac{[1-(1-\sigma_{\widetilde{p}})^N]}{N\sigma_{\widetilde{p}}}(1-\widetilde{p}) = \frac{[1-(1-\sigma_p)^N]}{N\sigma_p}(1-p) = U_p(\mu_{\widetilde{p}}, \mu_p; \sigma_{\widetilde{p}}, \sigma_p),$$

where $\sigma_{\widetilde{p}} + (M-1)\sigma_p = 1$.

Following BSW, the unique symmetric equilibrium entails a mixed strategy such that $\sigma_{\widetilde{p}} = \sigma_p = \frac{1}{M}$ and a price

$$p = \frac{M - \left(M + \frac{MN}{M-1}\right)(1-\frac{1}{M})^N}{M - \left(M + \frac{N}{M-1}\right)(1-\frac{1}{M})^N} \tag{1}$$

with expected payoffs for sellers and buyers, respectively,

$$\Pi_p\left(\mu_p; \sigma_p\right) = \left[1 - \left(1 - \frac{1}{M}\right)^N\right] p,$$

$$U_p(\mu_p; \sigma_p) = \frac{1-\left(1-\frac{1}{M}\right)^N}{\frac{N}{M}}\left(1-p\right).$$

In contrast to the previous literature, we now consider the possibility of deviating to bargaining. In particular, we allow one seller to deviate to a bargaining mechanism μ_b and $M-1$ sellers follow the price posting mechanism μ_p in equilibrium. The payoff of the deviating seller is

$$\Pi_b(\mu_b, \mu_p; \sigma_b, \sigma_p') = \left[1-(1-\sigma_b)^N\right]\eta$$

where σ_b represents buyers' probability of visiting the seller offering μ_b. All other non-deviating sellers are chosen with probability σ_p', which satisfies $\sigma_b + (M-1)\sigma_p' = 1$.

We focus on equilibrium with buyers using mixed strategy off the equilibrium path.[6] If

[6] This may appear to be a strong restriction, since it is well-known that in the directed search framework there are plethora of equilibria, where buyers play pure strategies on and off equilibrium path (see BSW). Bland and Loertscher (2012) allow buyers to play pure strategies on and off equilibrium path. They show

a buyer chooses a non-deviating price poster, his payoff is

$$U_p(\mu_b, \mu_p; \sigma_b, \sigma_p') = \frac{1 - \left(1 - \sigma_p'\right)^N}{N\sigma_p'}(1 - p).$$

If a buyer chooses a deviating bargainer, his payoff is

$$U_b(\mu_b, \mu_p; \sigma_b, \sigma_p') = \frac{1 - (1 - \sigma_b)^N}{N\sigma_b}(1 - \eta).$$

Then, all the sellers face the same BIC

$$U_b(\mu_b, \mu_p; \sigma_b, \sigma_p') = \frac{1 - (1 - \sigma_b)^N}{N\sigma_b}(1 - \eta) = \frac{1 - \left(1 - \sigma_p'\right)^N}{N\sigma_p'}(1 - p) = U_p(\mu_b, \mu_p; \sigma_b, \sigma_p').$$

Since p from (1) and η are taken as parameters by everyone off the equilibrium path, the BIC determines the mixed strategy σ_b. Then, we have the following proposition.

Proposition 2 *For any given M, N, and η, there exists a unique symmetric mixed strategy off the equilibrium path $\sigma_b \in (0, 1)$ in the buyers' subgame, if the following inequalities are satisfied:*

$$\frac{1}{N} < \frac{1 - p}{1 - \eta} < \frac{\frac{N}{M-1}}{1 - \left(1 - \frac{1}{M-1}\right)^N}.$$

Moreover, the strategy σ_b decreases with η.

Proof: *All proofs are in the Appendix.*

Larger η gives sellers higher prices, but the resulting buyers' selection probability, σ_b, is relatively small, making the deviation profitable. Note that in bargaining the price is independent of the market tightness.

Price posting is a robust equilibrium against deviations to bargaining if it is not profitable

that if buyers play a monotonic strategy, meaning that if a seller offers a lower price, buyers should put more weight on that seller off equilibrium path, then the unique equilibrium is a directed search equilibrium.

for sellers to deviate to bargaining, i.e., the SPC must hold.

$$\Pi_p(\mu_p;\sigma_p) = \left[1 - \left(1 - \frac{1}{M}\right)^N\right] p \geq \left[1 - (1-\sigma_b)^N\right] \eta = \Pi_b(\mu_b, \mu_p; \sigma_b, \sigma_p'). \tag{2}$$

For given M, N, and η, if $p = \eta$, then it is clear that $\sigma_b = 1/M$ and it never pays for sellers to deviate. When $p \neq \eta$, it is not obvious to pin down the analytical solution of σ_b (let alone unique) for which the SPC holds. Hence, we focus on sufficient conditions and have the following result.

Proposition 3 *For given M, N, and η, a sufficient condition for price posting to be a robust equilibrium against deviations to bargaining is given by*

$$\Pi_p(\mu_p;\sigma_p) = \left[1 - \left(1 - \frac{1}{M}\right)^N\right] \frac{M - \left(M + \frac{MN}{M-1}\right)(1-\frac{1}{M})^N}{M - \left(M + \frac{N}{M-1}\right)(1-\frac{1}{M})^N} > \eta.$$

There is a limiting market tightness, $\theta = N/M$, relative to the seller's bargaining power, η, for which sellers prefer price posting. Note that $\Pi_p(\mu_p;\sigma_p)$ is strictly increasing in θ. It is easier to support price posting as an equilibrium against bargaining when θ is large.

3.2 Auctions

Now consider the mechanism in JKK. All sellers choose an auction with a posted reserve price. Assume all sellers but one post μ_a where a candidate equilibrium reserve price r is offered, and the last seller deviates to $\mu_{\tilde{a}}$ where $\tilde{r} \neq r$. Let $\sigma_{\tilde{a}}$ be the probability with which buyers visit the deviating seller, and $\bar{\sigma}_a'$ represents the probabilities that buyers choose sellers who do not deviate. Then, the equilibrium outcome is

$$\Pi_{\tilde{a}}(\mu_{\tilde{a}}, \mu_a; \sigma_{\tilde{a}}, \sigma_a) = \max_{\tilde{r}}\{1 - (1-\sigma_{\tilde{a}})^N - (1-\tilde{r})N\sigma_{\tilde{a}}(1-\sigma_{\tilde{a}})^{N-1}\}$$

subject to the BIC

$$U_{\tilde{a}}(\mu_{\tilde{a}}, \mu_a; \sigma_{\tilde{a}}, \sigma_a) = (1-\tilde{r})(1-\sigma_{\tilde{a}})^{N-1} = (1-r)(1-\sigma_a)^{N-1} = U_a(\mu_{\tilde{a}}, \mu_a; \sigma_{\tilde{a}}, \sigma_a)$$

and $\sigma_{\tilde{a}} + (M-1)\sigma_a = 1$. Then, the unique symmetric equilibrium reserve price is

$$r = \frac{N-1}{N-1+(M-1)^2}, \tag{3}$$

with expected payoffs for sellers and buyers, respectively,

$$\Pi_a(\mu_a; \bar{\sigma}_a) = 1 - \left(1 - \frac{1}{M}\right)^N - (1-r)\frac{N}{M}\left(1 - \frac{1}{M}\right)^{N-1},$$

$$U_a(\mu_a; \bar{\sigma}_a) = (1-r)\left(1 - \frac{1}{M}\right)^{N-1}.$$

Now consider all sellers using the equilibrium auctions mechanism μ_a and one deviating to bargaining μ_b. Assume a buyer chooses a deviating seller with probability σ_b, and chooses a non-deviating auctioneer with probability σ'_a. If a buyer chooses a non-deviating auctioneer, his payoff is

$$U_a(\mu_b, \mu_a; \sigma_b, \sigma'_a) = (1-\sigma'_a)^{N-1}(1-r).$$

If a buyer chooses the bargainer who deviated from auction, his payoff is

$$U_b(\mu_b, \mu_a; \sigma_b, \sigma'_a) = \frac{\left[1-(1-\sigma_b)^N\right]}{\sigma_b N}(1-\eta),$$

where $\sigma_b + (M-1)\sigma'_a = 1$. Then, all sellers face the same BIC

$$U_a(\mu_b, \mu_a; \sigma_b, \sigma'_a) = (1-\sigma'_a)^{N-1}(1-r) = \frac{\left[1-(1-\sigma_b)^N\right]}{\sigma_b N}(1-\eta) = U_b(\mu_b, \mu_a; \sigma_b, \sigma'_a).$$

Since r given by (3) and η are taken as given, the BIC and $\sigma_b + (M-1)\sigma'_a = 1$ determine

$\sigma_b \in (0,1)$. Similarly, the upper bound of σ_b is 1.

Proposition 4 *For any given M, N, and η there exists a unique symmetric mixed strategy off the equilibrium path $\sigma_b \in (0,1)$ in the buyers' subgame, if the following inequalities are satisfied*

$$\frac{1}{N} < \frac{1-r}{1-\eta} < \frac{1}{\left(1-\frac{1}{M-1}\right)^{N-1}}.$$

Moreover, the strategy σ_b decreases with η.

Auctions are robust equilibria against deviations to bargaining if the SPC holds

$$\Pi_a(\mu_a;\bar{\sigma}_a) = 1-\left(1-\frac{1}{M}\right)^N-(1-r)\frac{N}{M}\left(1-\frac{1}{M}\right)^{N-1} \geq \left[1-(1-\sigma_b)^N\right]\eta = \Pi_b(\mu_b,\mu_a;\sigma_b,\sigma_a').$$

Proposition 5 *For given M, N, and η a sufficient condition for auction to be a robust equilibrium against deviations to bargaining is given by*

$$\Pi_a(\mu_a;\bar{\sigma}_a) = 1-\left(1-\frac{1}{M}\right)^N - \frac{(M-1)^2}{N-1+(M-1)^2}\frac{N}{M}\left(1-\frac{1}{M}\right)^{N-1} > \eta.$$

There is also an upper limit on market tightness for which deviations are not profitable. However, in contrast to the case of price posting, there exist M, N, and η such that $\eta < p$ and $\eta > r$. This is the case as the expected payoff of a seller running auctions is a convex combination of the price of 1 under multilateral match and the price of r in a bilateral match.

3.3 Hybrid Mechanism

Now we introduce a new multilateral pricing mechanism, a hybrid mechanism combining bargaining and auction. In particular, we consider μ_h as the following

$$\text{Hybrid (auction-bargaining): } \mu_h = \left\{ \begin{array}{l} p_n = \eta, \sigma_n = 1 \text{ if } n = 1 \\ p_n = 1, \sigma_n = \frac{1}{n} \text{ if } n > 1 \end{array} \right\}$$

In this mechanism, sellers bargain in a pairwise matching and exploit bidding in a multilateral matching. Assume that all sellers use the auction-bargaining mechanism μ_h. The expected seller's and buyer's payoffs are

$$\Pi_h\left(\mu_h;\bar{\sigma}_h\right)=\eta N\sigma_h\left(1-\sigma_h\right)^{N-1}+\left[1-\left(1-\sigma_h\right)^N-N\sigma_h\left(1-\sigma_h\right)^{N-1}\right]$$

and

$$U_h\left(\mu_h;\bar{\sigma}_h\right)=\left(1-\eta\right)\left(1-\sigma_h\right)^{N-1}$$

Given that η is exogenous, there are no variables directing search and buyers' search is completely random, i.e., $\sigma_h=\frac{1}{M}$ at equilibrium. The seller's expected profit then becomes

$$\Pi_h\left(\mu_h;\bar{\sigma}_h\right)=\eta\frac{N}{M}\left(1-\frac{1}{M}\right)^{N-1}+\left[1-\left(1-\frac{1}{M}\right)^N-\frac{N}{M}\left(1-\frac{1}{M}\right)^{N-1}\right].$$

Suppose all sellers use the auction-bargaining mechanism and only one seller deviates to pure bargaining. If a buyer chooses a non-deviating auction-bargainer, his payoff is

$$U_h\left(\mu_b,\mu_h;\sigma_b,\sigma_h'\right)=\left(1-\eta\right)\left(1-\sigma_h'\right)^{N-1},$$

where $\sigma_b+(M-1)\sigma_h'=1$. If a buyer chooses a deviated bargainer, his payoff is

$$U_b\left(\mu_b,\mu_h;\sigma_b,\sigma_h'\right)=\frac{\left[1-\left(1-\sigma_b\right)^N\right]}{\sigma_b N}\left(1-\eta\right).$$

Then, all sellers face the same BIC

$$U_h\left(\mu_b,\mu_h;\sigma_b,\sigma_h'\right)=\left(1-\eta\right)\left(1-\sigma_h'\right)^{N-1}=\frac{\left[1-\left(1-\sigma_b\right)^N\right]}{\sigma_b N}\left(1-\eta\right)=U_b\left(\mu_b,\mu_h;\sigma_b,\sigma_h'\right).$$

The BIC and $\sigma_b+(M-1)\sigma_h'=1$ together determine σ_b. Similarly, the upper bound of σ_b is 1.

Proposition 6 *For any given M, N, and η there always exists a unique symmetric mixed strategy off the equilibrium path $\sigma_b \in (0,1)$ in the buyer's subgame.*

This auction-bargaining equilibrium is robust if the SPC holds,

$$\begin{aligned}\Pi_h(\mu_h;\bar{\sigma}_h) &= \eta\frac{N}{M}\left(1-\frac{1}{M}\right)^{N-1}+\left[1-\left(1-\frac{1}{M}\right)^{N}-\frac{N}{M}\left(1-\frac{1}{M}\right)^{N-1}\right] \\ &\geq \left(1-(1-\sigma_b)^N\right)\eta = \Pi_b(\mu_b,\mu_h;\sigma_b,\sigma'_h).\end{aligned}$$

Proposition 7 *For any given M, N, and η a sufficient condition for auction-bargaining to be a robust equilibrium against deviations to bargaining is given by*

$$H(\mu_h;\bar{\sigma}_h) = \frac{1-\left(1-\frac{1}{M}\right)^N-\frac{N}{M}\left(1-\frac{1}{M}\right)^{N-1}}{1-\frac{N}{M}\left(1-\frac{1}{M}\right)^{N-1}} > \eta.$$

4 Comparing Mechanisms

In the previous sections, we have established sufficient conditions under which the proposed mechanisms are robust to deviations to bargaining. It is straightforward to show that for any given M and N, the following ranking holds

$$\Pi_h(\mu_h;\bar{\sigma}_h) > \Pi_a(\mu_a;\bar{\sigma}_a) > \Pi_p(\mu_p;\bar{\sigma}_p).$$

Let $\theta = N/M$. Define $(\widehat{\theta}_p, \widehat{\theta}_a, \widehat{\theta}_h)$ as the solutions to $\Pi_h(\mu_h;\bar{\sigma}_h,\widehat{\theta}_h) = \eta$, $\Pi_a(\mu_a;\bar{\sigma}_a,\widehat{\theta}_a) = \eta$, and $\Pi_p(\mu_p;\bar{\sigma}_p,\widehat{\theta}_p) = \eta$, respectively. Given the findings in Proposition 3, 5, and 7, we can establish the following result.

Proposition 8 *For any given M and $\eta \in (0,1)$, for all $\theta > \widehat{\theta}_p$, it never pays to deviate to bargaining under any of the mechanisms. When $\theta \in (\widehat{\theta}_a, \widehat{\theta}_p)$, only price posting is not robust to deviations. Finally, for $\theta \in (\widehat{\theta}_h, \widehat{\theta}_a)$, only auction-bargaining is robust to deviations.*

The number of buyers, N, needs to be large enough for all mechanisms to survive a deviation to bargaining. As the number of buyers decreases, only the auction-bargaining remains robust to such a deviation.

Similarly, it follows that for any given θ, we can rank the sufficient conditions under the different mechanisms by keeping θ constant and changing M and N by the same order. For any given M, there exist corresponding thresholds for the number of sellers for which the results in the previous proposition hold. This provides indications on, when the market gets larger, how the mechanisms can be supported as an equilibrium relative to deviations to bargaining. In particular, when θ gets larger, there is a larger range of values of η for which we can support any of the mechanisms as a equilibrium.

It is well-known that in directed search models, the bargaining power of agents is given by the extent of market tightness. We find that as M becomes smaller for any given θ, or as θ gets larger for any given M, it gets easier to support all mechanisms against deviations to bargaining.

5 Conclusion

In a finite market directed search framework, we explore the robustness of well-known mechanisms to deviations to bargaining. We find sufficient conditions under which the set, or a subset, of mechanisms that we consider in this paper are robust to a seller's deviation to bargaining. For any given seller's bargaining power, the set of mechanisms that can be supported as a robust equilibrium depends on market tightness. Finally, we show that there exist a range of market tightness values for which only the auction-bargaining mechanism proposed in this paper is robust to deviations to bargaining.

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Appendix: Proofs

Proof of Proposition 2

Rewrite the buyer's indifference condition as

$$\frac{1-p}{1-\eta} = \frac{(1-\sigma_b)\left[1-(1-\sigma_b)^N\right]}{(M-1)\,\sigma_b\left[1-\left(1-\frac{1-\sigma_b}{M-1}\right)^N\right]} \equiv \Theta(\sigma_b)$$

It is straightforward to show that $\Theta(\sigma_b)$ is strictly decreasing in σ_b and

$$\begin{aligned}
\lim_{\sigma_b \to 0} \Theta(\sigma_b) &= \frac{\frac{N}{M-1}}{\left[1-\left(1-\frac{1}{M-1}\right)^N\right]} \geq 1, \\
\lim_{\sigma_b \to 1} \Theta(\sigma_b) &= \frac{1}{N}.
\end{aligned}$$

Therefore, there exists a unique $\sigma_b \in (0,1)$ that keeps buyers indifferent between paying p and η. If $(1-p)/(1-\eta)$ is too big or too small, there are no mixed strategies off the equilibrium path. Hence, the condition follows. Since $(1-p)/(1-\eta)$ is increasing in η, σ_b is decreasing in η. ■

Proof of Proposition 4

Rewrite the buyer's indifference condition as

$$\frac{1-r}{1-\eta} = \frac{\left[1-(1-\sigma_b)^N\right]}{N\sigma_b\left(1-\frac{1-\sigma_b}{M-1}\right)^{N-1}} \equiv \Psi(\sigma_b)$$

One can easily show that $\Psi(\sigma_b)$ is strictly decreasing in σ_b and

$$\begin{aligned}
\lim_{\sigma_b \to 0} \Psi(\sigma_b) &= \left(1-\frac{1}{M-1}\right)^{1-N} > 1, \\
\lim_{\sigma_b \to 1} \Psi(\sigma_b) &= \frac{1}{N}.
\end{aligned}$$

Therefore, there exists a unique $\sigma_b \in (0,1)$ that makes buyers indifferent between paying r

and η. If is too big or too small, there is no mixed strategy off the equilibrium path, and hence the condition. Because $(1-r)/(1-\eta)$ is increasing in η, σ_b is decreasing in η. ■

Proof of Proposition 6

Rewrite the buyer's indifference condition as the following

$$1 = \frac{\left[1-(1-\sigma_b)^N\right]}{N\sigma_b\left(1-\frac{1-\sigma_b}{M-1}\right)^{N-1}} \equiv \Phi\left(\sigma_b\right).$$

It is easy to prove that $\Psi(\sigma_b)$ is strictly decreasing in σ_b and the following conditions satisfy.

$$\begin{aligned}
\lim_{\sigma_b\to 0}\Phi\left(\sigma_b\right) &= \left(1-\frac{1}{M-1}\right)^{1-N} > 1,\\
\lim_{\sigma_b\to 1}\Phi\left(\sigma_b\right) &= \frac{1}{N} < 1.
\end{aligned}$$

Hence, there always exists a unique $\sigma_b \in (0,1)$ that satisfies the buyer's indifference condition.

■